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Convective stability of a variable-viscosity fluid in a vertical slot

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Stability of convective motion generated by a lateral temperature difference in a variable-viscosity fluid contained in a vertical slot was studied experimentally. The experiments were carried out in a narrow tank 30 cm high × 11.5 cm deep × 2 cm wide with glycerin–water solutions of 70, 80 and 90% glycerine. The onset of instabilities was detected by means of shadowgraphs. As the critical temperature difference was exceeded, a regular array of convection cells appeared. The critical Rayleigh number showed a slight decreasing trend as the Prandtl number of the fluid was increased, with a concomitant increase in the viscosity variation. The critical wavenumber remained practically constant for the three solutions used. At large Rayleigh numbers, approximately seven times the critical value, boundary layers along the walls became unstable and the cellular convection pattern became disrupted.

1. Introduction

Thermal convection in fluids with temperature-dependent viscosity is of interest because of its importance in engineering and geological applications. The problem of a horizontal layer of variable-viscosity fluid being heated from below has been the subject of investigation by a number of authors. Torrance & Turcotte (1971), whose interest was in modelling the mantle convection, considered fluids with viscosity varying exponentially either with temperature or with depth. They solved by numerical means the two-dimensional, nonlinear equations appropriate for the problem. Defining the Rayleigh number Ra in terms of the kinematic viscosity evaluated at the mean of the top and bottom temperatures, they found that the Nusselt number $Nu \propto Ra^n$, with n not much different from that for the constant-viscosity fluids. Their results were qualitatively confirmed by experiments conducted by Booker (1976) and by Richter, Nataf & Daly (1983). Booker's experiments covered a range of viscosity ratios (the ratio of viscosity evaluated at the cold temperature to that at the hot temperature) up to 300; Richter *et al.* covered the range 10–10^5. These experiments show that the Nusselt number of the variable-viscosity fluid normalized by that for the constant-viscosity fluid at the same Rayleigh number varies by less than 20% from unity up to a viscosity ratio of 10^5.

The onset of thermal convection in a horizontal layer of variable-viscosity fluid has been examined by Stengel, Oliver & Booker (1982). For exponential and super-exponential viscosity variations, theoretical and experimental results show that the critical Rayleigh number rises slowly with the viscosity ratio, attains a maximum (3000 for glycerine), and then decreases. This is due to the fact that, at high viscosity ratios, convection first occurs in a layer of fluid immediately adjacent to the hot wall. Richter (1978) conducted experiments on the stability of convection rolls, which were induced initially by preferential heating in a variable-viscosity fluid. The results were similar to those found by Busse & Whitehead (1971) for constant-viscosity fluids.

The behaviour of a constant-viscosity fluid confined in a long, narrow tank subjected to lateral heating was first considered by Batchelor (1954). Since that time there have been a number of experimental, theoretical and computational investigations into this interesting problem. Many of these references have been cited in a recent paper by Lee & Korpela (1983) on computational results of fluids with Prandtl numbers up to 1000, and need not be repeated here. In the following, only the works which have direct bearing on ours will be discussed.

Elder (1965*a*, *b*) was the first to conduct detailed experimental studies of the characteristics of such flows using fluids with large Prandtl numbers. He found that, at low Rayleigh numbers, the flow was in the conduction regime, in which the temperature profile was linear and the velocity profile was cubic. As the Rayleigh number was increased, but still in the subcritical range, the convective motion became confined within boundary layers along the hot and cold walls, and the central core was nearly motionless and stably stratified in temperature. This was referred to as the boundary-layer regime. When the first critical Rayleigh number was exceeded, the unicellular convection broke down into a series of small convection cells, each sloping downward from the hot wall toward the cold wall. As the Rayleigh number was increased further, smaller counter-rotating cells would appear sandwiched in between the well-developed secondary cells. This was referred to by Elder as the tertiary motion. He also obtained the expression describing the velocity distribution across the tank, approximately taking into account the temperature stratification. Vest & Arpaci (1969) considered the stability of such convective motion in both the conduction regime and the boundary-layer regime. Their results showed that both basic flows were unstable. Quantitatively, their results are in slight error because a term was inadvertently left out in the linearized energy equation (Hart 1971).

Hart (1971) considered both theoretically and experimentally the more general case of the stability of convection in a sloping, narrow tank, with air and water as the fluid media. Linear stability analysis was made with respect to either transverse or longitudinal waves. The basic flow was that suitable for the boundary-layer regime using the experimentally obtained value of the temperature stratification. The results showed that, for water in a vertical tank, the transverse mode was the most unstable and the instability was in the oscillatory, or travelling-wave, mode. For sloping tanks with angles from the vertical $> 10°$, the longitudinal mode was found to be the most unstable.

To our knowledge, there has been only one set of experiments carried out in a vertical slot with variable-viscosity fluid in which the flow field was examined. Seki, Fukusako & Inaba (1978) performed experiments with transformer oil and glycerine and obtained streak photographs of the flow as the Rayleigh number was increased. They noted that the results for glycerine were different from those for the transformer oil because of the strong temperature dependence of the viscosity of glycerine. Their results will be discussed further in §3.

In this paper we study the stability of a fluid with temperature-dependent viscosity when it is subjected to differential heating across a narrow slot. Experiments are

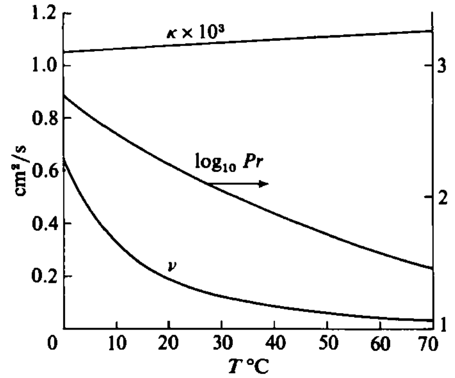


FIGURE 1. Variations of ν, κ and Pr with temperature for the 70 % glycerine solution.

performed in a tank with aspect ratio (height/width) of 15 using glycerine–water solutions with 70, 80 and 90 % glycerine. Critical conditions for the onset of instabilities are established; observations at high supercritical Rayleigh numbers are reported.

2. Apparatus and procedure

Experiments were carried out with glycerine–water solutions in a vertical tank 30 cm in height (H) × 11.5 cm deep × 2 cm in width (D). The two wide vertical walls (30 cm × 11.5 cm) were made of brass plates with passages milled in so that they could be maintained at constant temperatures by separate water baths. Two were used; one was a Lauda K-2/R refrigerated constant-temperature bath, and the other was a Haake bath with a Lauda IC-6 refrigeration unit to provide external cooling. The two narrow vertical walls were made of Plexiglas for flow-visualization purposes. In one interchangeable Plexiglas wall 30 thermocouples were placed 3 mm apart along the centreline. During all test runs the top of the tank was covered and the entire tank was insulated with styrofoam boards.

The flow rate through the water passages was such that the transit time was less than 2 s. As a result there was no detectable temperature drop between the inlet and the outlet ports. Thermocouples were placed in the water passages of the hot and cold walls. These readings are used as the plate temperatures. At the heat-flux values encountered with the high thermal conductivity of brass, the probable error incurred in the temperature difference was less than $2 \times 10^{-3}\, \Delta T$.

The test solution was prepared by placing the correct amounts of glycerine and water in a beaker and mixing thoroughly with a magnetic stirrer. After pouring the solution into the test tank, we waited until all motion had been damped out before starting the experiment. The physical properties of glycerin–water solutions can be obtained from Segur (1953). Values of kinematic viscosity ν, thermal diffusivity κ, and Prandtl number Pr are shown as functions of temperature for 70, 80 and 90 % glycerine solutions in figures 1, 2 and 3.

A total of 17 test runs were made; of these, a number were preliminary experiments carried out to determine the approximate value of the critical temperature difference ΔT_c for the onset of instabilities. Further experiments, run more carefully, were conducted to determine ΔT_c accurately. These latter tests were carried out with relatively large ΔT increments at first, then in successively smaller increments as the

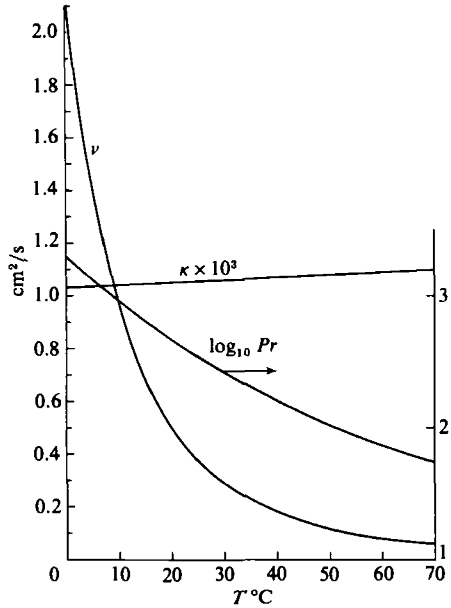


FIGURE 2. Variations of ν, κ and Pr with temperature for the 80 % glycerine solution.

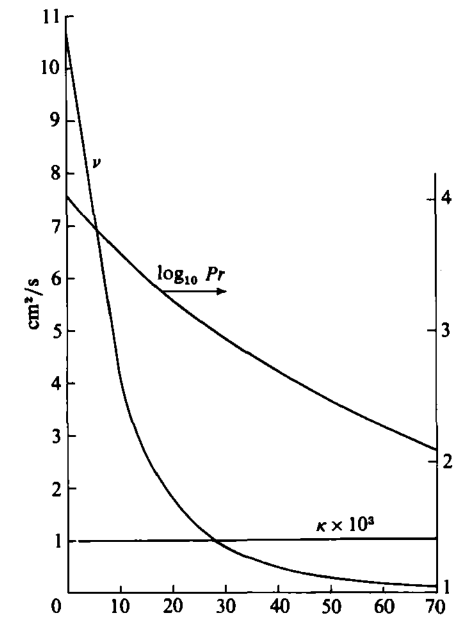


FIGURE 3. Variations of ν, κ and Pr with temperature for the 90 % glycerine solution.

critical value was approached. The actual test procedures for the 70, 80 and 90 % glycerine solutions are presented in table 1. Whenever possible, ΔT values were obtained by increasing the hot-wall temperature and decreasing the cold-wall temperature by the same amount. As can be seen from table 1, this was accomplished for all adjustment steps except the last three steps for the 90 % glycerine solution. By that stage, the cold bath was at its limit; increases in ΔT could only be obtained

Glycerine 70 %		Glycerine 80 %		Glycerine 90 %	
ΔT	Number of adjustments	ΔT	Number of adjustments	ΔT	Number of adjustments
± 1 °C	6	± 3.5 °C	2	± 7.5 °C	1
± 0.5 °C	1	± 5 °C	1	± 4.5 °C	1
± 0.25 °C	4	± 1 °C	2	± 3 °C	2
$\Delta T_c = 16.0$ °C		± 0.5 °C	4	± 2 °C	4
Possible error = ± 1.5 %		$\Delta T_c = 31.8$ °C		$+3$ °C	3
		Possible error = ± 1.5 %		$\Delta T_c = 49.1$ °C	
				Possible error = ± 3 %	

TABLE 1. Test procedure to obtain ΔT_c

by increasing the temperature of the hot wall. Adjustments in ΔT were made every hour, which corresponded to the thermal-diffusion time. The possible error in the determination of the critical ΔT due to the discrete steps of temperature increase for each solution is also listed in Table 1.

Flow visualization was usually accomplished by use of the shadowgraph technique, although we did obtain several dye pictures. A slide projector placed approximately 3 m away from the test tank provided the light source. At the onset of instabilities, hot fluid would be drawn into the top of a convection cell while cold fluid would be drawn into the bottom cell. Through variations in refractive index, the cell became visible on a shadowgraph.

3. Experimental observations

A convective flow was generated in the tank as soon as a temperature difference was imposed across the tank. The motion was in the form of a single-cell flow, upward along the hot wall and downward along the cold wall. As the ΔT was increased further, but was still in the subcritical regime, the boundary layer along the cold wall became thicker than that of the hot wall because the difference in viscosity became more pronounced. The temperature in the core of the fluid became stably stratified, as in the case for constant-viscosity fluids.

To visualize the flow pattern after the onset of instabilities, we tried three different methods: dye tracing, reflecting particles, and shadowgraphs. With the dye-tracing technique, it was difficult to determine the point of onset. The other drawback was that the solution became too dark to be used again. In spite of these disadvantages, dye tracing does give a dramatic picture of the convection cells, as shown in figure 4, which is a photograph of the 90 % glycerine solution at a supercritical ΔT of 55.8 °C. Dark-green dye crystals were dropped into the fluid from the top, and they slowly dissolved. At first, the dyed solution moved up and down the tank in the boundary layers along the two walls. The difference in the thicknesses of these two boundary layers is clearly exhibited in the photo. After about 15 min, the dyed solution was slowly drawn into the convection cells and showed four fully developed cells in a distance of approximately 10 cm.

We next tried introducing small amounts of Rheoscopic concentrate AQ-1000 into the solution for visualization purposes. Cell patterns similar to those obtained by Elder (1965*a*) were observed. However, it was difficult to arrive at an optimum concentration for photographic purposes. Furthermore, the effects of their presence

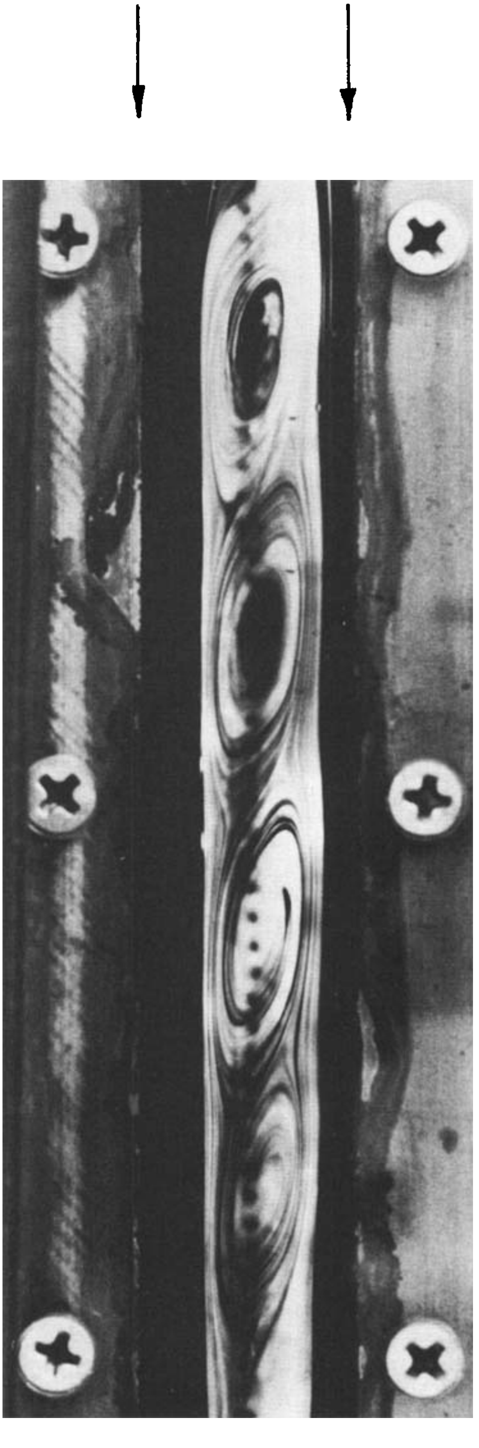


FIGURE 4. Dye-trace picture obtained for the 90% glycerine solution at $\Delta T = 55.8$ °C. The critical ΔT is 49.1 °C. Boundary layers along both walls are completely opaque through the presence of dye. It is noted that the hot boundary layer on the right is considerably thinner than the cold one on the left. The screws are 5.08 cm between centres, and the arrows indicate the locations of the two walls, which are 2 cm apart.

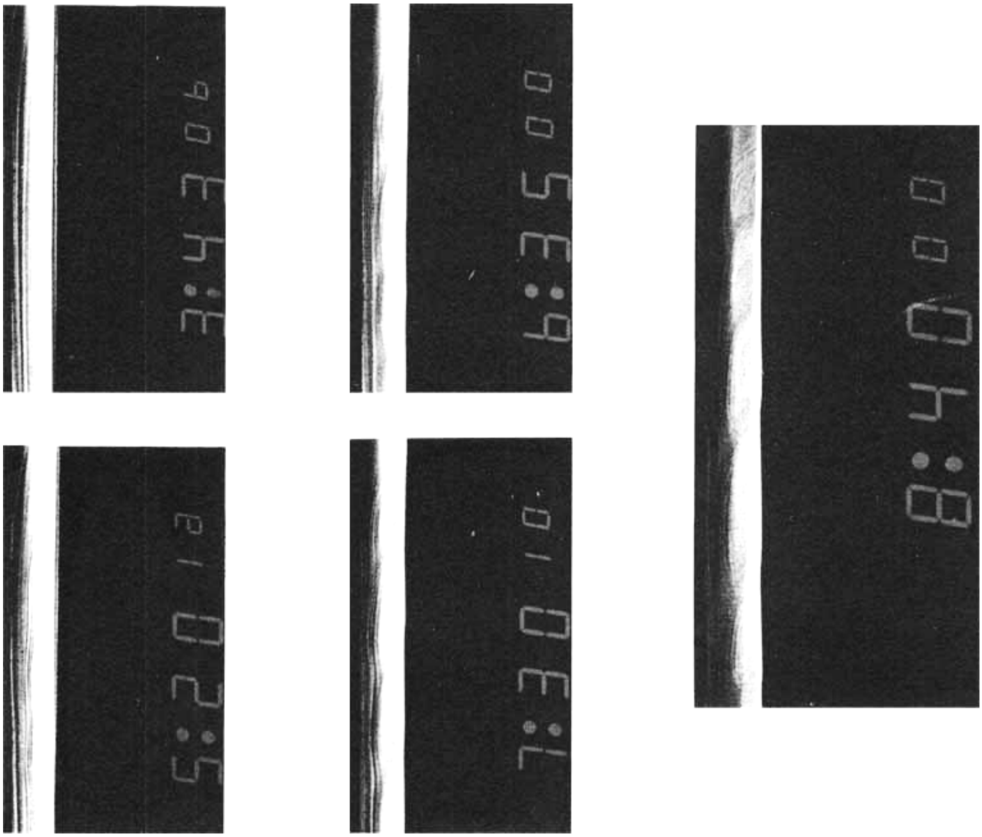
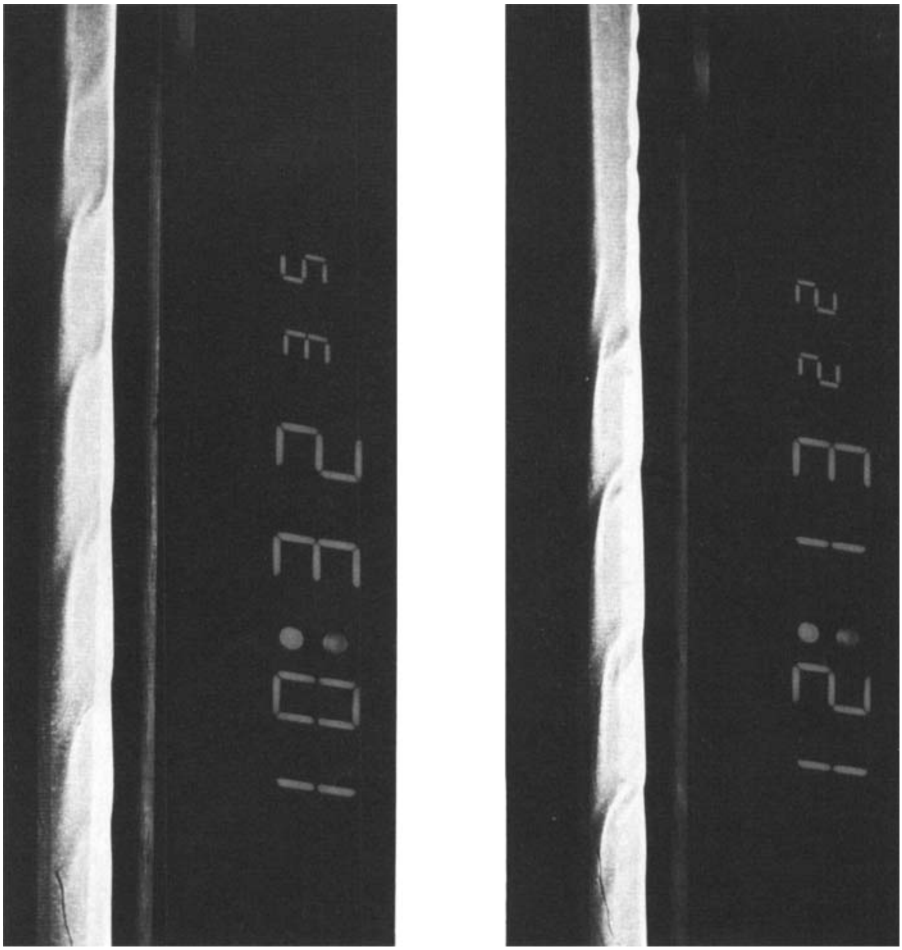


FIGURE 5. Shadowgraphs for the 80 % glycerine solution near the marginal state. The ΔT values for these five cases are 30.2, 33, 35.3, 39.3 and 45 °C. Temperature increases were made at 3.30, 6.00, 7.00 and 8.00 p.m.

on the viscosity and its variations are not known and may be important. We therefore chose the shadowgraph technique.

In figure 5, we present a series of five shadowgraphs obtained for the 80 % glycerine solution. The time when the photo was taken and the ΔT across the tank are shown below each picture. The test was started in the morning; by 3.30 p.m. the temperature difference across the tank reached 25.5 °C. The critical ΔT was found to be 31.8 °C in another test. At 3.30 p.m. the water baths were adjusted to give a supercritical temperature of 33 °C. The thermal response of the system was such that it took approximately 20 min for the hot wall to reach the preset temperature in the bath. The first shadowgraph was taken at 3.43 p.m. showing the onset of instabilities at a ΔT of 30.2 °C and increasing. The impulsive increase of 7.5 °C in ΔT evidently triggered the onset at a slightly lower temperature. The width of the tank appearing in the shadowgraph is somewhat reduced from the actual size owing to the bending of light rays away from the hot wall (on the right). This effect can be seen as a bright band in the photo. The starting cell was in the form of a teardrop bordered on the left by a series of dark and light lines, indicating alternating layers of fluid with different temperatures. A millimetre scale was placed along the cold wall for length measurements.

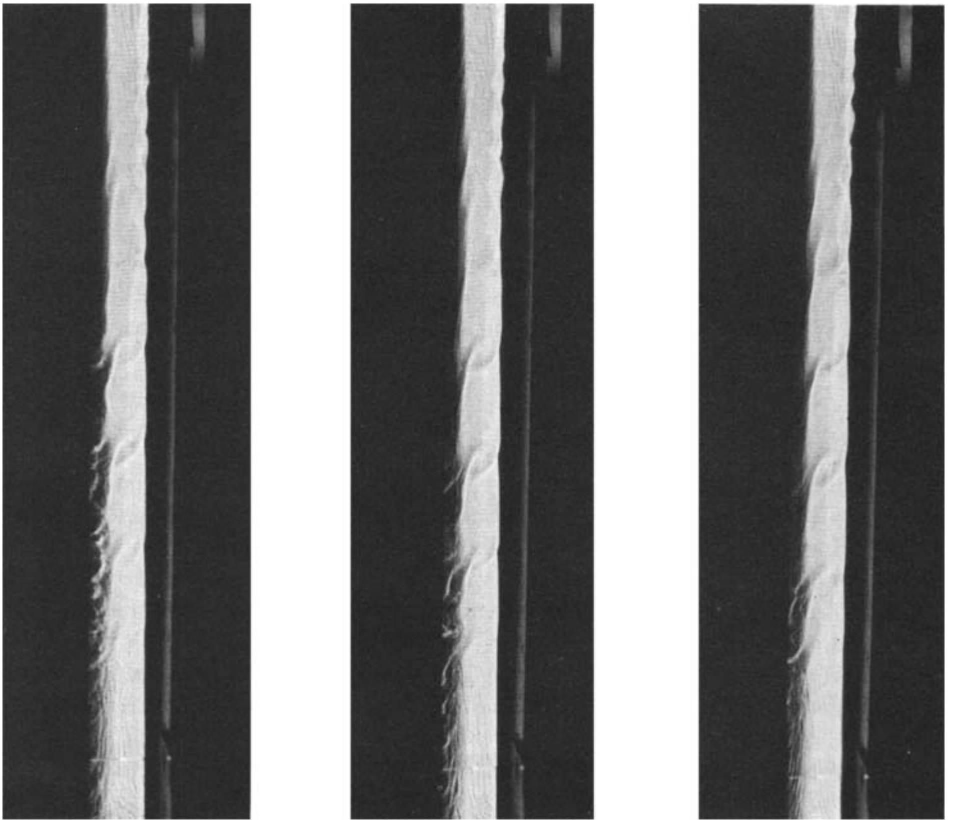
The temperature difference of 33 °C was maintained till 6.00 p.m. The shadowgraph



(a) (b)

FIGURE 6. Shadowgraphs for the 70% glycerine solution at supercritical Rayleigh numbers. (a) 2.5 Ra_c; (b) $5.4Ra_c$.

at 5.20 p.m. showed three cells, in a steady state for $\Delta T = 33$ °C. The ΔT was increased to 35.3 °C at 6.00 p.m. and its effect can be seen in the shadowgraph taken at 6.35 p.m. There were four cells, and the convection within the cells was more vigorous, as evidenced by the presence of more dark and bright bands within each cell. As the ΔT was further increased by 4 °C, the lateral boundaries of the cells were being pushed towards the cold wall, as can be seen in the 7.30 p.m. shadowgraph. The final, close-up view of the convection cells was taken after the ΔT had been increased to 45 °C. The visible portion of the slot was increased by 15%. There were five cells in view, and the circulation in each cell was clearly exhibited by the shadows. The flow of the cold fluid toward the hot wall at the bottom of each cell caused the appearance of the scallop-edge of the dark shadow along the hot wall. Although the ΔT is almost $1.5\Delta T_c$ the Rayleigh number is only $1.3Ra_c$ because the mean temperature in the tank was increasing as well. It is noted here that the



(a) (b) (c)

FIGURE 7. Shadowgraphs for the 70% glycerine solution at the supercritical Rayleigh number of $7.4Ra_c$. The pictures were taken at (a) 1.39 p.m; (b) 1.41 and (c) 1.45.

Rayleigh number is defined in terms of fluid properties evaluated as the mean temperatures:

$$Ra = \frac{g\alpha \,\Delta T\, D^3}{\nu\kappa},$$

in which g is the gravitational acceleration, α the thermal-expansion coefficient, ν the kinematic viscosity, and κ the thermal diffusivity.

To study the flow behaviour at supercritical Rayleigh numbers to $7Ra_c$ we carried out a set of experiments with 70% glycerine because of its lower ΔT_c. At $2.5Ra_c$ (figure 6*a*), smaller convection cells began to appear at the interfaces of the existing convection cells. This was referred to by Elder (1965*a*) as the tertiary-flow regime. At $5.4Ra_c$ (figure 6*b*), the tertiary cells became well developed. Furthermore, the boundary layer along the hot wall (on the right) became unstable at approximately 7 cm from the top of the tank, and a wave-like motion started there and rolled upward. As a consequence of this wave motion, the two uppermost cells were destroyed. The Rayleigh number, based on the length of 23 cm and viscosity corresponding to the temperature of the hot wall, was 6.56×10^9, well above the value of 4×10^8 observed by Eckert & Soehnghen (1951) for the onset of instabilities along a heated plate in air.

In figure 7, we present a sequence of three shadowgraphs taken at still higher Rayleigh numbers of $7.4Ra_c$. The photos were taken within 6 min, at 1.39, 1.41, and 1.45 p.m. The boundary layer along the cold wall became unstable as well. The instability waves occurred in spurts; they would start at mid-height and progress downward, as shown in the photos. The same phenomenon occurs at the hot wall, but not quite as vigorously. The Rayleigh number, based on the length of 15 cm and viscosity corresponding to the cold-wall temperature, was 5.45×10^8, and was comparable to the critical value observed by Eckert & Soehnghen in air.

Elder (1965*b*) studied turbulent free convection in tanks with aspect ratios in the range 10–30. These were carried out at much higher Rayleigh numbers than the ones we were using. In this study all convection cells were destroyed and the flow was dominated by a turbulent core in the centre of the tank.

4. Quantitative results and discussion

Vertical temperature stratification in the tank was studied for the 70 and 90% glycerine solutions by obtaining centreline temperature profiles in the mid-height of the tank. The vertical temperature gradient β, rendered non-dimensional by $\Delta T/D$, is shown as a function of the Rayleigh number in figure 8. As seen from the figure, the value of β is insensitive to the concentration of glycerine in the range 70–90%, and it stays constant at a mean value of 0.021 in the range $10^5 \leqslant Ra \leqslant 5 \times 10^5$. Both Elder (1965*a*) and Hart (1971) showed that β attains its asymptotic value for $Ra \geqslant 10^5$. Hart's data for water yielded a value of $\beta = 0.04$ for the same aspect ratio as our tank. This value agrees with that obtained by Eckert & Carlson (1961) for air. Elder obtained a value of 0.033 for paraffin and 0.037 for 100 cSt silicone oil. He also detected a slight decrease in β for paraffin, which was attributed to the effect of variable viscosity. In our experiments, however, this trend was not evident.

Before presenting the critical conditions obtained by the experiments, a discussion of the possible sources of error is in order. The solution was obtained by mixing correct amounts of glycerine and water, which were measured using an Ohaus scale accurate to ± 0.05 g. Since the samples required were in hundreds of grams, the possible error is less than 0.1%. Possible error in temperature determination is ± 0.1 °C, which would cause an error of ± 1% in the kinematic viscosity for the 90% solution at room temperature. Less concentrated solutions would incur smaller errors. Both α and κ can be considered constant within the temperature range of interest.

Since the tank width D appears in the third power in the Rayleigh number, it would be the major source of error. The tank was fabricated with a width of 20 ± 0.127 mm, or ± 0.625%. Together with the possible errors in ΔT (listed in table 1) and ν, the possible error in the Rayleigh number for the worst case is ± 6%. The wavenumber was determined by the average wavelength of the convection cells. The wavelengths of individual cells may vary ± 10% from the average value.

The experimentally determined critical conditions, together with the property values used for their evaluation, are presented in table 2. Two separate tests were carried out for the 70% solution to check the repeatability of the experiments. The critical Rayleigh numbers and wavenumbers did agree within the experimental error. For the 70% and 80% solutions we were able to keep the mean temperature T_m at room temperature. For the 90% solution, however, it was about 10 °C higher. The viscosity ratios varied from 2.1 to 27.2, and the mean Prandtl number from 160 to 720.

The critical Rayleigh numbers show a slight downward trend as the glycerine

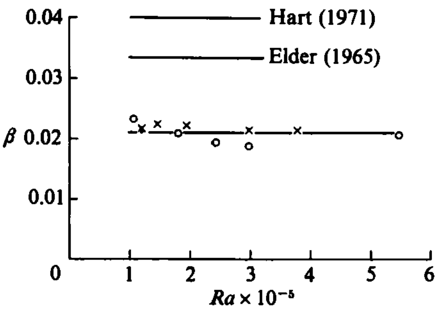


FIGURE 8. Vertical temperature gradient β as a function of Rayleigh number: $\times$, 90% glycerine; $\bigcirc$, 70% glycerine.

	Glycerine %			
	70	70	80	90
T_h (°C)	30.4	30.2	38.6	57.9
T_c (°C)	14.4	12.8	6.7	8.8
ΔT_c (°C)	16.0	17.4	31.9	49.1
T_m (°C)	22.4	21.5	22.7	33.4
ν_c/ν_h	2.05	2.15	6.70	27.20
ν_m (cm²/s)	0.170	0.175	0.425	0.720
κ_m (cm²/s × 10³)	1.075	1.075	1.05	1.00
α_m (°C⁻¹ × 10⁴)	5.65	5.65	6.1	6.2
Pr_m	158	163	405	720
$Ra_c \times 10^{-5}$	3.88	4.10	3.42	3.32
k_c	3.1	3.1	2.9	3.4

TABLE 2. Experimental critical conditions and property values

concentration is increased. Seki *et al.* (1978) presented streak photographs of the convection pattern in transformer oil ($Pr = 480$) and glycerine ($Pr = 12500$). Their experiments were conducted in a tank of the same dimensions as ours. If we assume that figures 3(*b*) and 4(*b*) in their paper are streak photographs obtained when the convective flow is only slightly supercritical, then we have the following results:

Pr	Ra_c
480	2.96×10^5
12500	1.04×10^5

When these two data points, together with our four, are shown on a semi-log plot in figure 9, they do exhibit some degree of self-consistency.

The experimentally determined critical wavenumbers stayed sensibly constant with an average value of 3.1. No attempt has been made to obtain such values from the photos given by Seki *et al.* (1978) because it was difficult to determine the width of the slot.

A linear stability analysis has been made for a variable-viscosity fluid heated laterally across a slot of infinite height (Thangam & Chen 1984). In this case, the basic state is in the conduction regime and there is no vertical temperature stratification. The viscosity of the fluid is assumed to vary exponentially with temperature. Using the best exponential fit to the viscosity data within the temperature range of interest

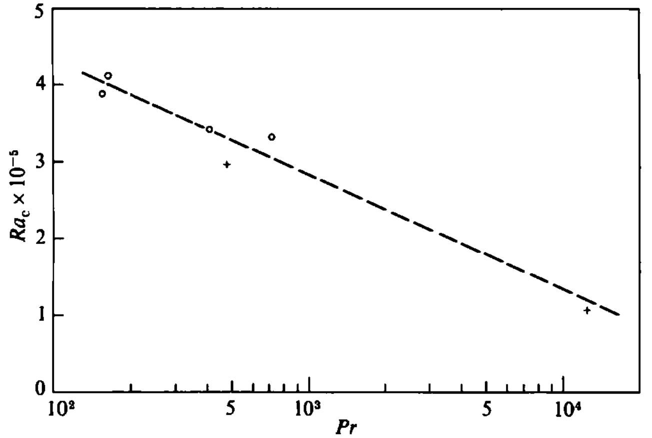


FIGURE 9. Experimental critical Rayleigh number as a function of the Prandtl number: ⊙, present data; +, data from Seki, Fukusako & Inaba (1978); -----, a line drawn through the experimental points.

	Glycerine %		
	70	80	90
$Ra_c \times 10^{-5}$	1.507	1.709	1.396
k_c	2.387	2.459	2.238
σ_1	1.33×10^3	1.127×10^3	0.736×10^3
Period of oscillation (s)	17.58	21.24	34.13

TABLE 3. Predicted critical conditions based on conduction solution

for each of the solutions tested, we have found that the onset of instability is in the oscillatory mode. The critical Rayleigh number, critical wavenumber, and the frequency of oscillation are listed in table 3. Comparing the values of k_c and Ra_c with those listed in table 2, we see that the experimental critical parameters are higher than the theoretical ones. The critical wavenumbers in both cases do not vary much for the three solutions. The average k_c obtained theoretically is 2.36 and that obtained experimentally is 3.1. Since in the secondary-flow regime the convection cells are confined within the central core (see figure 4), which is quite a bit narrower than the width of the tank, it is not surprising that the experimental wavelengths are smaller than the theoretical ones. The experimental critical Rayleigh number is approximately twice that of the theoretical value. This discrepancy is due to the difference in the basic states between the theoretical and experimental cases. If the boundary-layer effects were taken into account, a better agreement between the experimental observations and the theoretical predictions should be expected.

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